Solutions

Sol. 1→
$$\frac{\partial J}{\partial W} = \frac{1}{m} \sum_{i=1}^{m} \left(Wx^{(i)} + b - y^{(i)} \right) \cdot x^{(i)}$$

Sol. 2→
$$\frac{\partial J}{\partial b} = \frac{1}{m} \sum_{i=1}^{m} \left(Wx^{(i)} + b - y^{(i)} \right)$$

Sol. 3→ $W = 1$, $b = 3$, $\alpha = \frac{1}{9}$

$$\hat{y}^{(i)} = Wx^{(i)} + b$$
$$= x^{(i)} + 3$$

$$\hat{y}^{(1)} = x^{(1)} + 3 = 0 + 3 = 3$$
$$\hat{y}^{(2)} = x^{(2)} + 3 = 1 + 3 = 4$$
$$\hat{y}^{(3)} = x^{(3)} + 3 = 2 + 3 = 5$$
$$\hat{y}^{(4)} = x^{(4)} + 3 = 3 + 3 = 6$$

$$J(W,b) = J(1,3) = \frac{1}{2(4)} \left[\left(\hat{y}^{(1)} - y^{(1)} \right)^2 + \left(\hat{y}^{(2)} - y^{(2)} \right)^2 \right.$$
$$+ \left(\hat{y}^{(3)} - y^{(3)} \right)^2$$
$$\left. + \left(\hat{y}^{(4)} - y^{(4)} \right)^2 \right]$$

$$= \frac{1}{8} \left((3-4)^2 + (4-7)^2 + (5-8)^2 + (6-9)^2 \right)$$

$$= \frac{1}{8} \left(1^2 + 3^2 + 3^2 + 3^2 \right)$$

$$= \frac{1}{8} (28) > 0.5$$

$$W = 1 - \frac{1}{9} \left(\frac{\partial J}{\partial W} \right) = 1 - \frac{1}{9} \left(\frac{-9}{2} \right) = 1 + \frac{1}{2} = 1.5$$

$$b = 3 - \frac{1}{9} \left(\frac{\partial J}{\partial b} \right) = 3 - \frac{1}{9} \left(\frac{-5}{2} \right) = 3 + \frac{5}{18} \approx 3.28$$